Induced Drag and Lift of Wing by the Piecewise Continuous Kernel Function Method

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Abstract

THE piecewise continuous kernel function method is extended to compute induced drag on three dimensional wings in subsonic flow The method was used to compute induced drag and lift on a wide number of wings, and the results compared very well with other existing methods The method was applied to compare induced drag and lift of nontwisted planar swept forward and back wings with the equivalent M or W type configuration It was found that the spanwise load distribution and the induced drag are in fluenced by the sweep angle of the lifting surface. Fur thermore, it was found that the spanwise pressure distribution of the M wing type offers some advantages that can be used to improve the configuration performance characteristics as one wishes

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Unlike a two dimensional airfoil, the finite lifting wing does experience a downstream force (drag due to lift) in a subsonic inviscid flow One way to compute this resistance is by examining the wake at remote points behind the trailing edge. If the wing is observed moving through the fluid at rest, it is noted that an amount of mechanical work is done on the fluid owing to the passing wing. This mechanical work ap pears as a wake behind the traveling wing The nature of this wake consists of a sheet of trailing vortices parallel to the flow direction where the spanwise distribution of the wake cir- culation is affected by the pressure distribution on the lifting surface At points far downstream, the motion produced by the trailing vortices becomes two dimensional in a plane perpendicular to the flow direction (the so called Trefftz plane) Although the wake is assumed to remain flat in ac cordance with the small perturbation hypothesis, in fact, some rolling up and downward displacement occurs This rolling up can be shown to influence the loading that is only of third order in angles of attack (for a mono-lifting surface)

Following Multhopp,[1] the expression for the induced angle of attack due to the trailing vortices behind the wing (the Trefftz plane) can be stated as

$$\alpha_i(y) = -\frac{1}{8\pi}\int_{-s}^{s}\frac{c(\eta)c_l(\eta)}{(y-\eta)^2}\mathrm{d}\eta \tag{1}$$

where $\alpha_i\ (y)$ is the spanwise distribution of the induced angle of attack; s is the semispan of the wing; $c(\eta)$ is the local chord at the η spanwise location; and $c_l(\eta)$ is the sectional lift coefficient

The induced angle of attack of Eq (1) causes an induced drag due to lift as follows:

$$C_{D_i} = \frac{1}{S}\int_{-S}^{S} c_l(y)c(y)\alpha_i(y)\mathrm{d}y \tag{2}$$

where C_{D_i} is the induced drag coefficient and S is the lifting surface area

Examining Eqs (1) and (2) reveals that the computation of the induced drag is independent of the pressure chordwise distribution on the lifting surface. Thus, it is the spanwise distribution of the lift $c(\eta)c_l(\eta)$ that mostly influences the induced drag

For computing the spanwise pressure load on the lifting wing, one has to solve the integral equation that relates the downwash to the pressure distribution on the lifting surface For solving this singular integral equation[2] to get the pressure distribution across the wing, we applied the piecewise con tinuous kernel function method (PCKFM) The main ideas of the PCKFM were reported in several previous papers [2,3] The PCKFM combines the rapid convergence characteristics of the kernel function method with the ability of the finite element method to treat pressure discontinuities without having to determine their exact form Thus, the PCKFM is very suitable for computing the pressure distribution on lifting surfaces with geometrical discontinuities as deflected controls, chords discontinuities such as those existing at the root of swept wings, or at wing surface break points such as those existing at M type wing configurations

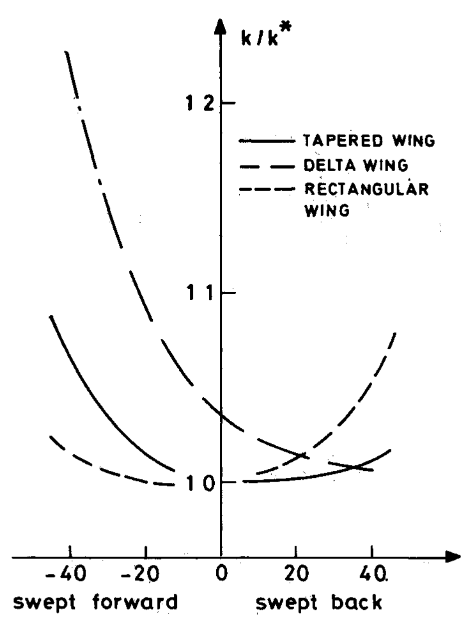

Fig 1 Variation of the k parameter vs sweep angle for three lifting surfaces ($A\!\!\!R = 4$; $M = 0\ 8$; and $\lambda_{1/4}$ is the forward quarter chord of the configuration)

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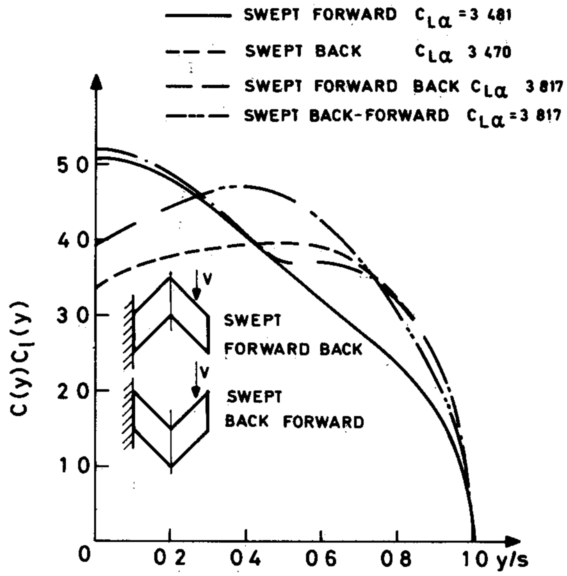

Fig 2 Spanwise lift distribution for four constant chord wings ($\mathcal{R}=4$; $M=0$ 8; and sweep angle $\lambda=45$ deg)

The pressure distribution in each of the boxes formed by the PCKFM can therefore be represented by a series of con tinuous orthogonal polynomes as follows:

$$p(\xi\ \eta)=\sum_{l=1}^{nb}\sum_{j=1}^{ns}\sum_{i=1}^{nc}A_{m\,l}W(\eta)P_j(\eta)w(\xi)p_i(\xi)/c(\eta) \quad (3)$$

where $A_{m,l}$ represents a scalar coefficient at the lth box; ns and nc are the number of spanwise and chordwise pressure polynomials, respectively; and $m=(j-1)^*nc+i$ $P_j(\eta)$ and $p_i(\xi)$ are the orthogonal pressure polynominal type assumed in the spanwise and chordwise directions, respectively The polynomes $P_j(\eta)$ and $p_i(\xi)$ are orthogonal to the weight functions $W(\eta)$ and $w(\xi)$, respectively

Applying the PCKFM to solve the wing integral equation, one gets the pressure polynominal coefficients $A_{m\,l}$ for each box l Integrating the pressure along the chord, one obtains the spanwise load distribution

$$c(\eta)c_l(\eta)=\sum_{l=1}^{nb}\sum_{j=1}^{ns}A_{m\,l}W(\eta)P_j(\eta)H \quad (4)$$

where

$$H=\sum_{i=1}^{nc}\int_{c_{\mathrm{LE}}}^{c_{\mathrm{TE}}}w(\xi)P_i(\xi)\mathrm{d}\xi/c(\eta)$$

$$=\int_{c_{\mathrm{LE}}}^{c_{\mathrm{TE}}}w(\xi)p_l(\xi)\mathrm{d}\xi/c(\eta) \quad (5)$$

Owing to the orthogonality of the chordwise polynomes $p_i(\xi)$ the $c(\eta)c_l(\eta)$ distribution is influenced only by the zero order chordwise polynome $P_l(\xi)$ [see Eq (5)]

Substituting Eq (4) for Eq (1) results in the following expression for the induced angle of attack:

$$\alpha_i(y)=-\frac{H}{8\pi}\sum_{l=1}^{nb}\sum_{j=1}^{ns}A_{m\,l}\int_{-S}^{S}\frac{W(\eta)P_j(\eta)}{(y-\eta)^2}\,\mathrm{d}\eta \quad (6)$$

The double pole singularity of the integral in Eq (6) is from the same type as the integral occurring in the wing integral To accomplish these spanwise singular integrals the spanwise integration is divided into four regions, as was stated in Ref 2

The PCKFM was coded to run on the IBM 3081D at the Technion, using the IBM double precision mode A con vergence study of the induced drag computed by various numbers of chordwise and spanwise pressure ploynomes indicates that one obtains a converged solution for a three chordwise by three spanwise pressure polynomes per box (selected as default value for the computer code)

The induced drag can be expressed as $C_{D_i}=kC_L^2$ where the k parameter (constant for small angle of attack) is an in dicator of the induced drag developed for a prescribed value of lift The k parameter is normalized by the theoretical value for an elliptic spanwise load distribution, hence by $k^*=1/\pi\mathcal{R}$ A comparison study was conducted on a hyperbolic constant chord wing, $\mathcal{R}=4$ in incompressible flow (the planform is defined in Ref 5) The PCKFM gives $k/k^*=1$ 0387, compared with 1 0457 computed by the VLM (Ref 4) and 1 0380 computed by NPL, NLR, or BAC methods [5] A similar comparison study was done for the Warren 12 swept wing $\mathcal{R}=2\sqrt{2}$ in incompressible flow (the planform is defined in Ref 5) The PCKFM gives $k/k^*=1$ 01, compared with 1 0107 computed by the VLM (Ref 4) and 1 01 computed by the NPL, NLR or BAC methods [5]

The sweeping of the lifting surface either forward or backward changes the load distribution along the span thus influencing the k parameter of the wing Figure 1 displays the variation of the k parameter vs sweep angle for three various lifting surfaces These three configurations have the same aspect ratio ($\mathcal{R}=4$) and spans, and their wing shapes are rectangular, delta, and a tapered wing with a taper ratio $c_{\mathrm{tip}}/c_{\mathrm{root}}=1/3$ Figure 1 pointed out that the variation of the k parameter is not symmetric with respect to the angle of sweep owing to the way the spanwise load distribution changes with respect to the sweep angle of the wing

Figure 2 shows a comparison study of spanwise distribution of lift on an M and W type wing compared with swept forward or backward rectangular wings All four con figurations have the same aspect ratio $\mathcal{R}=4$; the same sweep angle, $\lambda=45$ deg; and are flown at $M=0$ 8 A higher lift coefficient (C_{L_α}) and the concentration of lift toward the root chord of the wing are some of the advantages of the spanwise pressure distribution of the M wing type that are clearly displayed in Fig 2

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